General Information about the Class

Mittwoch, 2. März 2016 11:35

Office hours are Thurs, 11 am
registration via email

35% Midterm 26.4.
35% Endterm 29.6
30% Homeworks

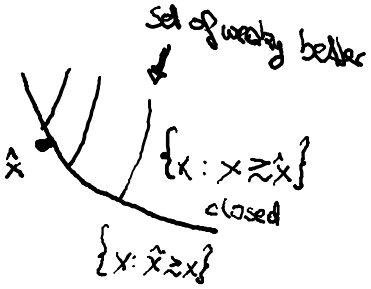
open classnotes
we will not solve everything
2½ perfect → 1
½ half → 2
What is the role of reputation.

Preferences

Mittwoch, 2. März 2016 11:38

Continuity math.

for continuity the set needs to be closed!

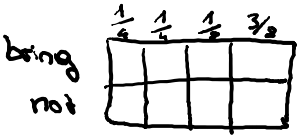


we also have that pref. need to be/ conti

rational, continuous, independend, real

Today we'll construct the numbers for utility

being not

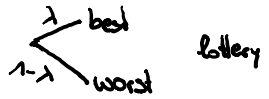


$u(\text{worst}) = 0$

$u(\text{best}) = 1$

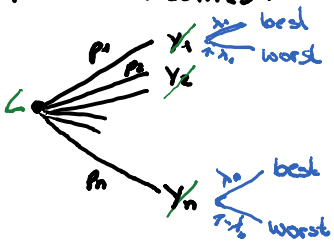
one does not always have a definitley worst and best outcome. If set of outcomes is finite we always have best and worst

$u(y_i) = \lambda \Leftrightarrow [y_i] \sim \lambda[\text{best}] + (1-\lambda)[\text{worst}]$

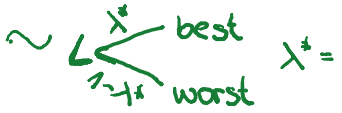


lottery

for n outcomes:



we can do this because of independence



$\lambda^* =$

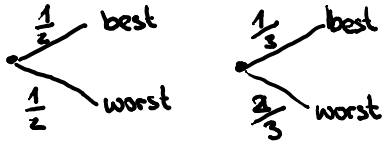
because of reduction axiom

$u(L) = \lambda^*$

$$u\left(\sum_{i=1}^{n} p_i [y_i]\right) \overset{!}{=} \sum_{i=1}^{n} p_i \, u(y_i)$$

This is what we want to show in the End

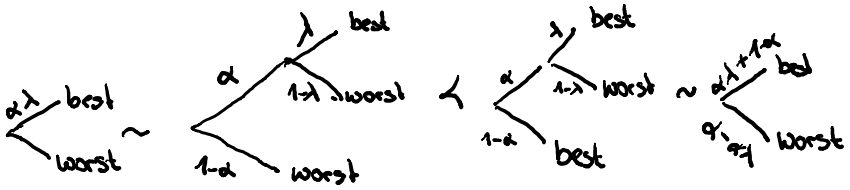
first we look at:



intuitivley we prefer the left lottery

the tricky thing is math. proof.

combine it such that



for the example above: $\frac{1}{3} = \alpha\lambda$

nächstes Beispiel: $\frac{1}{2} = \alpha\lambda + 1 - \alpha \Rightarrow \alpha = \frac{5}{6}$

	S_1	S_2
T	1	0
B	0	1

$$\{T\} = BR\Big(\overbrace{\tfrac{2}{3}[S_1] + \tfrac{1}{3}[S_2]}^{\prec_S}\Big) = BR\Big(\big(\underset{S_1}{\tfrac{2}{3}}, \underset{S_2}{\tfrac{1}{3}}\big)\Big)$$

(he prefers this notation)

B is stupid given $\prec_S$ (stupid = not rational)

T is rational given $\prec_S$

$\frac{2}{3}[T] + \frac{1}{3}[B]$ can be rational if beliefs $(\frac{1}{2}, \frac{1}{2})$

BR to $(\frac{1}{2}, \frac{1}{2})$ is any combination: $\Delta\{T,B\} = BR((\frac{1}{2}, \frac{1}{2}))$

	S_1	S_2	min	maxmin
T	5	5	5	→ without specifying beliefs.
B	6	-100	-100	→ max min Theory

only interesting in desicions.
Does not make a lot of sense in games

Max min: regret

	S_1	S_2
T	0.01	0.01
B	0	100

→

	S_1	S_2
T	0	99.99
B	0.01	0

minimize Max. regret!

randoizing is not a good idea usually

randomization under subj exp. utility "rare" event. → ?

"Game Theory"

Mittwoch, 2. März 2016 12:42

(action set)

$a \in A$ is strictly dominated by $b \in A$ if $u(a, s_i) < u(b, s_i) \; \forall_i$

$\Updownarrow \; u(a, \mathcal{L}_s) < u(b, \mathcal{L}_s) \; \forall \mathcal{L}_s \in \Delta\{s_1 \dots s_m\}$

Def. a is stupid if $\forall \mathcal{L}_s \; \exists b$ s.t. $u(a, \mathcal{L}_s) < u(b, \mathcal{L}_s)$
(never BR)

a is dominated:
$u_i(a_i, a_{-i}) < u_i(b_i, a_{-i}) \quad \forall a_{-i} \in A$
bei weakly $\leq$, und manche strictly

	S_1	S_2
T	6	3
B	2	1

B is dominated by T

	S_1	S_2
T	4	0
M	0	4
B	1	1

B is strictly domiated by $\frac{1}{2}[T] + \frac{1}{2}[M]$

• a strictly dominated $\Rightarrow$ a stupid
$\overset{?}{\Leftarrow}$ ✓ proof later

Desicions:

actions, outcomes, form beliefs, establish utilities $\overset{vNM}{\Longrightarrow}$ max. Exp utili

$$\underbrace{u\left(\tfrac{1}{2}[T] + \tfrac{1}{2}[B], p\right)}_{\in \underset{\sigma \in \Delta A}{\operatorname{argmax}} u(\sigma, p)} = \tfrac{1}{2} \underbrace{u(T, b)}_{= u(B,p) = u\left(\frac{1}{2}[T] + \frac{1}{2}[B], p\right)} + \tfrac{1}{2} u(B, p)$$

$$\Rightarrow C(\sigma) \subseteq \{s_i \text{ s.t. } s_i \in \underset{s'}{\operatorname{argmax}} \, u(s', p)\}$$

support — set of all pos. weights; pure best responses

	S_1	S_2
T	2	0
B	2	1

S

a_1 is weakly dominated by a_2 if $u(a_1, p) \leq u(a_2, p) \; \forall p \in \Delta S$
s.t. $u(a_1, p') < u(a_2, p')$ some $p' \in \Delta$

Games

Mittwoch, 2. März 2016 15:00

Games are like desicions, but often Theory of beliefs

n players

A_i: action set of player i

$u_i : \times_{i=1}^{n} A_i \to \mathbb{R}$ vnM

$\{(a_1, a_2, a_3, \dots a_n) \text{ st } a_i \in A_i \forall i\}$

add: common knowledge of rationality

	L	R
T	2,0	0,1
B	3,1	1,0

$\Rightarrow$ B, L

↳ iterated elimination of stupid strategies

Game: "Say a number between 0 and 100. You win if your number is $\frac{2}{3}$ of average"

$A_i = \{1, \dots 100\}$ (randomly allocate among winners)

n players

outcomes: win or not win

100 is strictly dominated by $\sum_{a=1}^{99} \frac{1}{99}[a]$ (to be proven)

↳ eliminate

↳ 99 strictly dominated

↳ eliminate

↳ 1 outcome of common knowledge rationality.

	L	R
T	2,2	0,2
B	2,0	1,1

→ cannot eliminate weakly dominated strategy). It kills the efficient outcome

rationalizable: if it can be played in a under common knowledge of rationality.

decisions: strict dom. $\Leftrightarrow$ stupid

2 person games: $\Leftrightarrow$

n-person games: $\Rightarrow$ if beliefs are independent $\Leftarrow$ if correlated beliefs

	L	R
T		
B		

X

	L	R
T		
B		

Y

	LX	RX	LY	RY
T				
B				

$A_1 \{T, B\}$ $A_2 \{L, R\}$ $A_3 \{X, Y\}$

	L	R
T	2,2	0,0
B	0,0	1,1

9.3.

"Battle of the sexes"

Box \ Ballet	Box	Ballet
Box	3,1	0,0
Ballet	0,0	1,3

what typically happens:
girls choose ballet, guys choose mixed

→ there are no stupid desicions

3 NE: (X,B) (B,X) $\left(\frac{1}{4}[X] + \frac{3}{4}[B], \frac{3}{4}[X] + \frac{1}{4}[B]\right)$

Email game

A \ A	A	B
A	M,M	1,-L
B	-L,1	0,0

$(1-p)$

$L > M > 1$

$p < \frac{1}{2}$

B \ A	A	B
A	0,0	1,-L
B	-L,1	M,M

p

player 1 knows which game is played, has an incentive to tell P2

automatic communication: 1 informs 2 about game s.t. inf. does not arrive with prob. $\varepsilon > 0$

2 informs 1 about info recieved, if no event then stop

1 2

2 1

:

will this ε change the game? If $\varepsilon = 0$ we have common knowledge

Information set: Player 1: knows A, plays A, also if she doesn't get an answ.

{situation you can be in} Player 2: doesn't get email, plays A because $p < \frac{1}{2}$

P1 knows it's B

given 1 gets no message back. It could be that — 1st message didn't arrive ε
                                                — 2nd message didn't arrive $(1-\varepsilon)\varepsilon$

$$P(\text{mess. didn't arrive} \mid \text{no message back}) = \frac{\varepsilon}{\varepsilon + (1-\varepsilon)\varepsilon} = \frac{1}{2-\varepsilon} > \frac{1}{2}$$

⇒ BR is A

P2 got 1 Message with B. doesn't get mess. back

⇒ BR is A

by iteration ⇒ always chose A

⇒ almost common knowledge is not common knowledge

Information transmission, basically like no information

The non-intuitive thing is infinite communication

Only 1 email is enough

credible threats need to be subgame perfect.

there are no subgames in this game!!

Nash Equilibria

Mittwoch, 9. März 2016 12:29

Best Response: $BR_i(\sigma_{-i}) = \{\sigma_i : u_i(\sigma_i, \sigma_{-i}) \geq u_i(a_i, \sigma_{-i}) \; \forall a_i \in A_i\}$ (pure)

A_i = pure strategies

$\sigma_i \in \Delta A_i \quad u_i(\sigma_1 \dots \sigma_n) = u_i(\sigma)$

$\sigma_{-i} = (\sigma_1, \dots, \sigma_{i-1}, \sigma_{i+1}, \dots, \sigma_n)$

Nash Equilibrium:

$\sigma^* NE \Leftrightarrow \sigma_i^* \in BR(\sigma_{-i}^*) \; \forall i$

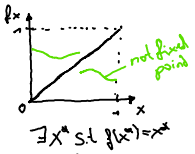
In finite games (finite action sets) there are equilibria.

No equilibrium in "say the highest number" (or number closest to zero)

if $|A_i| < \infty \; \forall i \Rightarrow \exists NE$

$NE \Leftrightarrow \sigma^* \in \bigtimes_{i=1}^n BR_i(\sigma_{-i}^*)$

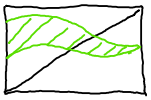
(Wakutani Fixed Point Theorem)



$\exists x^*$ s.t. $f(x^*) = x^*$

f is continuous

$f: [0,1] \to [0,1]$



$\exists x^*$ s.t. $x^* \in f(x^*)$

many fixed point.

construction of NE is a fixed point.

Sometimes it's hard to find, → trial & error

no hierachy in NE.

Equilibria are not necessarily efficient (like PD)

Nash must be in cross-product of BR $\forall i$ and σ^* is fixed point of R (correspondence)

Game Theory is not necessarily egoistic: that depends on outcome evaluation

Proof of statement above Kakutani

$r: B \to\!\!\to B$ (correspondence)

s.t. i) B compact, convex $\subseteq \mathbb{R}^n$

ii) $r(b) \neq \emptyset \; \forall b \in B$

iii) $r(b)$ convex $\forall b \in B$

iv) r is a closed graph

$\Rightarrow \exists$ fixed point of r, $\exists b^*$ s.t. $b^* \in r(b^*)$

convex: we can mix also between mixed strategies

Proof: $\forall$ finite games $\exists NE$

$\sigma^* NE \Leftrightarrow \sigma_i^* \in BR_i(\sigma_{-i}^*) \; \forall i$

$\Leftrightarrow \sigma^* \in \bigtimes_{i=1}^n BR_i(\sigma_{-i}^*) \Leftrightarrow : \sigma^*$ fixed point of r

$r(\sigma) := \bigtimes_{i=1}^n BR_i(\sigma_i^*)$

$r(T) = \{T\}$ we always need to map into sets

$B = \bigtimes_{i=1}^n \Delta A_i$

σ is what everybody does

σ_i is what a specific person does

The proof so far is only about one shot games

finite games → Action set is finite! And finite # of players **Most games are actually finite**

To find BR one does NOT need to look at mixed strat! only pure! find them and mix them. mixed is BR

ad i:) one can find a BR to others actions ✓ because A is finite there is always a best of them.

iii) $r(b)$ is a set, it needs to be convex. ✓

i) closed : S action S ✓

bounded ✓

convex ✓

iv) graph shows the functions

continious, so there are no jumps if we reach the limit, this is what the closed graph shows

comes from linearity, hardest part to proof, but intuitive?

Application: Proof for 2 person game (decisions/n-person with correlated beliefs)

to show: $\bar{a}_1$ strictly dominated $\Leftrightarrow$ $\bar{a}_1$ never best response if game is finite

Proof: $\bar{u}_1(\sigma) := u_1(\sigma) - u_1(\bar{a}_1, \sigma_2)$ to proof: $\exists \sigma_1^*$ s.t. $u(\sigma_1^*, \sigma_2) > u(\bar{a}_1, \sigma_2) \; \forall \sigma_2$

$\bar{u}_2(\sigma) := -\bar{u}_1(\sigma)$

$\exists \hat{\sigma} = NE$ of $(\bar{u}_1, \bar{u}_2)$ game, as it is finite.

$\Rightarrow \overline{u}_1(\hat{\sigma}) = u_1(\hat{\sigma}) - u_1(\overline{\sigma}_1, \sigma_2) > 0$

as $\overline{u}_1(\sigma_1, \hat{\sigma}_2) > 0 \quad \forall \sigma_1 \in BR_1^*(\hat{\sigma}_2)$

$\Rightarrow \overline{u}_1(\hat{\sigma}_1, \sigma_2) > 0 \quad \forall \sigma_2$ because NE and #2 hurts #1

$\Leftrightarrow u_1(\hat{\sigma}_1, \sigma_2) > u_1(\overline{\sigma}_1, \sigma_2) \quad \forall \sigma_2$

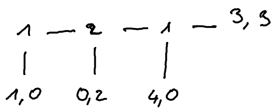
Time

Dienstag, 15. März 2016 14:35

Perfect Information Game

so for we had two ways of solving
- rationalizable
- NE

"always down", there is no final period



"backward induction"

time matters (?)

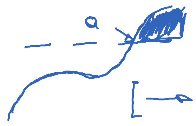
Theorem: (Debreu 1952) (n players; finite)

$A_i \subset \mathbb{R}^m$, $A_i \neq \emptyset$, compact, convex $\forall i$
u_i continuous in a, quasi concave in a_i } $\exists$ NE

this is not fast,
This could be used to proof existence of NE
in Cournot w/ contin. demand

quasi concave means: $\{x \text{ s.t. } f(x) > a\}$ convex

concave function
convex set.
→ quasi concave



[—→ x s.t. f(x) > a

Theorem Glicksberg (1952)
$A_i \subseteq \mathbb{R}^m$, $A_i \neq \emptyset$ compact $\forall i$
u_i cont. in a
n players } $\exists$ NE

Zero-sum games

u zero-sum game w/ $n=2$, so $u_2 = -u_1$

Theorem: if σ^* NE $\Rightarrow u_1(\sigma^*) = \max_{\sigma_1} \min_{\sigma_2} u(\sigma)$

$u_1: A \to \mathbb{R}$
$A = A_1 \times A_2$
$\sigma_1 \in \Delta A_1$; $\sigma_2 \in \Delta A_2$

$\sigma_1^* \in \arg\max_{\sigma_1} \min_{\sigma_2} u_1(\sigma)$

Note: $u_1(\sigma^*) = \min_{\sigma_2} \max_{\sigma_1} u_1(\sigma)$

Note $\nexists$ NE $\Rightarrow u_1(\hat\sigma) = u_1(\sigma^*)$

To show:
$\max_{\sigma_1} \min_{\sigma_2} u_1(\sigma) \overset{?}{\leq} \min_{\sigma_2} \max_{\sigma_1}(\sigma)$ ($\geq$)

$u_1(\bar\sigma) = \max_{\sigma_1} \min_{\sigma_2}$ $\quad u_1(\hat\sigma) = \min_{\sigma_2} \max_{\sigma_1}$

$u_1(\bar\sigma) \leq u_1(\bar\sigma_1, \hat\sigma_2) \leq u_1(\hat\sigma)$

Example for max min

	L	R	min
T	2,2	0,0	0
B	0,0	1,1	0

transform to zero-sum

		L	R
1/3	T	2,-2	0,0
2/3	B	0,0	1,-1

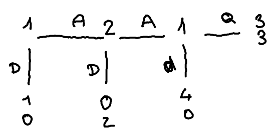
mix to make the other one indifferent

$\frac{1}{3}[T] + \frac{2}{3}[B]$ maxmin strategy

σ^* NE $u_1(\sigma^*) \overset{=}{\geq} u_1(\sigma_1, \sigma_2^*) \overset{=}{\geq} \min_{\sigma_2} \max_{\sigma_1} u_1(\sigma) \overset{=}{\geq} \max_{\sigma_1} \min_{\sigma_2} u_1(\sigma) \overset{=}{\geq} u_1(\sigma^*)$ (from above; check: max is better than any other) $\Rightarrow$ equal signs at all steps
$= -u_2(\sigma^*)$

q.e.d.

Back to game above:



A strategy is a plan for every node

(,)
A or D, a or d

Normal form:

	D	A
Dd	1,0*	1,0
Da	1,0*	1,0
Ad	0,2	4,0
~~Aa~~	~~0,2~~	~~3,3~~

$(\mu[Dd] + (1-\mu)[Da], \lambda[A] + (1-\lambda)[D])$
NE if $\lambda \leq \frac{1}{4}$
$\mu \in [0,1]$

Aa ← strictly dominated by $\lambda[Da] + (1-\lambda)[Ad]$ $\forall$ small $\lambda > 0$

↑ we cannot eliminate A because weak dominance

rationalizability ⇒ (1,0) (0,2) (4,0)
NE ⇒ (1,0)

Ultimatum game

I : $x \in [0,1]$

II $f : [0,1] \to \{Y, N\}$

Sequentially Rational SPE

Dienstag, 5. April 2016 13:19

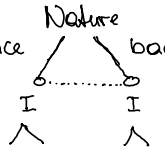
Repeating what we did before easter.

Time as a concept, learning

How to model nice vs. not nice

rationalizable: Outcome that is achieved by noone making non-rational choices. (iterated elimination of strictly dominated strategies)

imperfect information

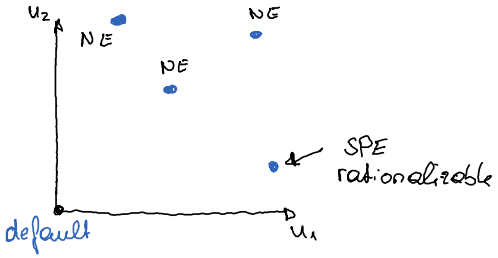


time comes in

→ sequentially rationalizable = rationalizability at all nodes/info-set

seq. rational ⇏ backward induction

Back to ultimatum game:



$x \in [0,1]$ feasible

assume: u_1 is strictly increasing in x

seq. rat. is $x=1$

profile = I: $x=1$
II: y if $u_2(x') > 0$ $\forall x'$

P1 has 4 options.
P2 has 2 possib. at 4 Points → 4^2 strategies

SPE: for every node the game from now on is a NE.
Subgame perfect Nash Equilibrium

One-shot-Deviation principle

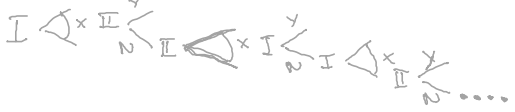
BR_i can be checked at each node specifically (only 1 deviation)
if:
- Game is finite (does not go on forever)
- ∃ discounting

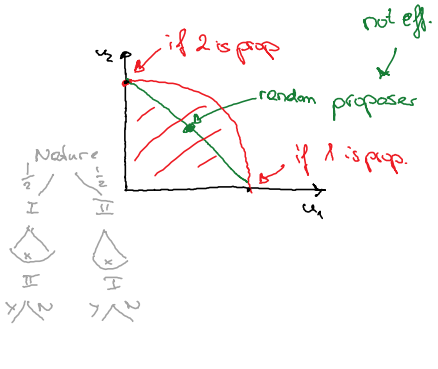
Bargaining $n=2$

1) Ultimatum game: take it or leave it

2) Random poposer (i) w/ proposer randomly chosen (equally)

3) Alternating offers bargaining (Rubenstein)





in this game time comes in, every future payoff is multiplied by δ
if δ is close to 0 → ultimatum game, the more interesting case is δ close to 1

Introduce new notation:

$\overline{V_1}$ highest pay for P1 in SPE

$\underline{V_1}$ lowest ——"——

$u_1(x) = x$
$u_2(x) = 1-x$

$$\underline{V_1} \geq 1 - \delta_2 \overline{V_2} \overset{*}{\geq} 1 - \delta_2 (1 - \delta_1 \underline{V_1}) \Rightarrow \underline{V_1} \geq \frac{1-\delta_2}{1-\delta_1\delta_2}$$

$$\overline{V_1} \leq 1 - \delta_2 \underline{V_2} \leq 1 - \delta_2 (1 - \delta_1 \overline{V_1})$$

$$\overline{V_2} \overset{*}{\leq} 1 - \delta_1 \underline{V_1} \qquad \overline{V_1} \leq \frac{1-\delta_2}{1-\delta_1\delta_2}$$

by def: $\underline{V_1} \leq \overline{V_1}$

$\Rightarrow \underline{V_1} = \overline{V_1} = \frac{1-\delta_2}{1-\delta_1\delta_2}$ ⇒ only 1 SPE

As this is a common knowledge game SPE is implemented immediately

for δ close to 1 : 50/50 split.

convex cake

- Eq. solution should be on the line
- very close to the point on the line, straight line
- because it's not a question of numbers but "more" or "less"

→ look for the tangency that splits if 1:1

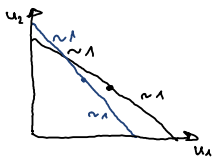
Nash bargaining + Alternady (?) Offers Bargaining.

Mittwoch, 6. April 2016 11:37

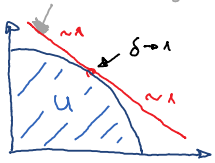
(I)

→ from yesterday:

$V_1 = \frac{1-\delta_2}{1-\delta_1\delta_2}$



(like the budget set in CD)



find the tangent, that splits the bargaining set equally

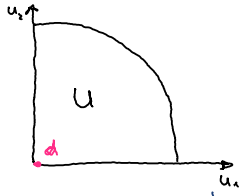
Bargaining set needs to be convex

could be exam question!!

$\max_{u \in U} u_1 u_2$ (Cobb Douglas)

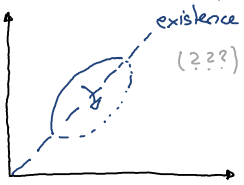
Existence proof, we do not proof uniqueness (for now?)

(II) Nash Bargaining → Axiomatic



(i) pareto efficient
(ii) depends on preferences (not utility, as utility is a construct, pref. are internal)
(iii) independence of lables if o/w no difference. (if pref. are symmetric, outcome is equal)!
(iv) independence of irrelevant alternatives.

(iii)



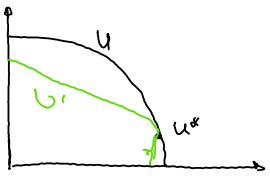
$u^* \in \underset{u \in U}{\operatorname{argmax}} (u_1 - d_1)(u_2 - d_2)$

even if we change utility represent. max is the same

so ⇐ is simple ✓

other direction ⇒ ?

(iv)



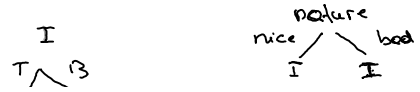
u^* is opt in U, $U' \subseteq U$, $u^* \in U' \Rightarrow u^*$ opt. U'

„⇒"

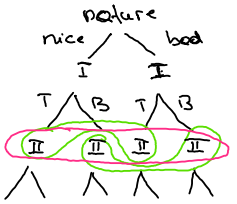
change. $u^N \to (\frac{1}{2}, \frac{1}{2})$ change scales
$d \to (0,0)$ substract d from everything
} s.t. same pref.

Information

incomplete utility is unknown (don't know if the other one is good or bad)
imperfect: move is unknown



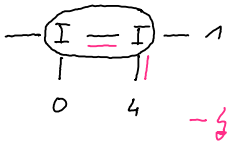
imperfect
(for simultaneous games)



II observes I
II does not know I's move

incomplete (+imperfect)

game: tree
nodes (node: player, set of actions
set of other nodes that are indistinguishable)



$\frac{1}{2}$ not poss.

Assume: perfect recall!

→ in this case randomize!
to prevent nature from hurting me.

not possible!

Practice: Play coordination game 2 times.

	L	R
T	2,2	0,0
B	0,0	1,1

SPE outcomes of PA: $4, 2, 3, \frac{4}{3}, \frac{8}{3}, \frac{5}{3}$,

NE: $2, 1, \frac{2}{3}$
outcomes

Punishment
want to implement (B,L) for whatever reason

2 rounds: II L if (T,L) so if I decides: $2 + \frac{2}{3}$ } doesn't work
if not: $0 + 2$

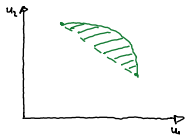
3 rounds: I decides: $2 + \frac{2}{3} + \frac{2}{3}$ } works
not: $0 + 2 + 2$

	1st	2nd		3rd	
I	B	T	$\frac{1}{3}[T] + \frac{2}{3}[B]$	T	mix
II	L	L	$\frac{1}{3}[L] + \frac{2}{3}[R]$	L	mix
if		(BL) 1st	not (B,L) 1st	(B,L) 1st	o/w

⇒ in finite repeated games
- have to play NE if one-shot in last round
- not necessarily earlier

Finitely Repeated Games

Dienstag, 12. April 2016 13:20

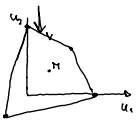


In this lecture we treat mixing like any other strategy
We are not averse to mixing → Expected utility
We (don't?) have free disposal.

Finitley repeated games.

repeat k times, sum utilities.
SPE as solution concept • in last round we will play NE

	H	D
H	-1,-1	3,0
D	0,3	2,2



3 regimes ← close to end, cooperation, punishment

	C	D
C	3,3	0,5
D	5,0	1,1

• (D,D) in last round.
• cannot enforce (C,C) in k-1.
→ (D,D) in each round in SPE

NE does not require (D,D) in all rounds
NE could be: • (D,D) in all rounds except if (C,C) in round 1, in that case C,C always.
• (D,D) in eq. path except if (D,C)^{k-1} then (C,C) → $(k-1)\cdot 0 + 3 \leq k$ if $k \geq 3$
NE ⇏ (D,D) on eq. path

Theorem: Game has unique NE (x^*, y^*)
① ∃ SPE, (x^*, y^*) in each round
② other outcomes may exist if $\bar{v}_i < u_i(x^*, y^*)$ $\forall i$

add "irrelevant", "stupid" alternative

	C	D	S
C	3,3	0,5	0,0
D	5,0	1,1	$\frac{1}{2}$,0
S	0,0	0,$\frac{1}{2}$	0,0

• last round: (D,D) → 1
→ cooperate up to $(k-4)^{th}$ round.
(S,D) forever if (C,D) in one of the first (k-4) rounds
(both players punish)
o/w (D,D) (in last 4 rounds) I need 4 rounds to punish again of 2 with a loss of $\frac{1}{2}$
→ need the last couple of rounds to ensure earlier cooperation.

Max Punishment

pure strategies: $\bar{v}_i := \min_{a_{-i}} \max_{a_i} u_i(a)$

mixed strategies $\bar{v}_i := \min_{\alpha_{-i} \in \Delta A_{-i}} \max_{a_i} u_i(a_i, \alpha_{-i})$ $\left(= \max_{\sigma_i \in \Delta A_i} \min_{\alpha_{-i} \in A_{-i}} u_i(\alpha_i, \alpha_{-i})\right.$ if game is finite$\left.\right)$

s.t. $\bar{V}_i \geq \bar{v}_i$

Theorem: if $v > \bar{u}_i$ $\forall i$ and ∃ NE (x^*, y^*) s.t. $u_i(x^*, y^*) > \bar{v}_i$ $\forall i$
then ∃ NE of k-times game w/ average payoff ≈ v if k suff. large and v feasible.

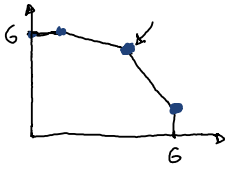
Theorem: If $v > \bar{v}_i$ and ∃ NE (x^*, y^*) $\forall i$ ∃ NE (x^i, y^i) s.t. $u_i(x^i, y^i) < u_i(x^*, y^*)$
then ∃ SPNE if k suff. large.

Punishment, Deviation in finetly repeated games

Mittwoch, 13. April 2016 11:35

where do equilibria come from. If there is one eq. it's clear but with many, which do you play. Conventions
In battle of the sexes predict coordination because of conventions
NBS is what society thinks is good

2 Times battle of the sexes.



	B	X
B	3,1	0,0
X	0,0	1,3

max $u_1 u_2$

$\to a^{(1)}\ a^{(2)} \dots a^{(m)} \to x^{(m+1)} \dots x^{(k)}$ } cycles of length n (= #players)
$x^1 \dots x^n$
$y^1 \dots y^n$
$u_i(x^i) \geq u_i(y^i)$
punish by replace ith

no deviations
$\geq$ 2 dev.
(we only react if 1 person deviates)

j deviates in round r
$y^{(r+1)} \dots y^{(k)}$

$a^{(m)} \in \times_{i=1}^{n} A_i$
$x^{(m)} \in \times_{i=1}^{n} \Delta A_i$ $x^{(s)}$ NE of stage game

After $(m+1)^{st}$ round only NE $\to$ no deviations there.
We have to look at one-sided deviation up until m.

if implementing NE: minimax player i for $y^{(j)}$ for sufficiently many $j \in \{r+1, \dots, k\}$

if implementing SPNE: y^j NE of stage game for suff. — // —

Example BoS $a^{(j)} = (X,B)$ $\forall\ j \leq m \leq \frac{k}{2}$
$x^j = (B,B), (X,X)$
$y^j = (X,X)$ if P1 deviates

Example HD $a^j = (D,D)$
$x^j = (D,H), (H,D)$
$y^j = (D,H)$ if P1 deviates

Subgame perfection? Once we reach $x^{(m+1)}$ NE in each round.

∞ repeated Games

Example: PD
assume we don't know if current round is last

	C	D
C	3,3	0,5
D	5,0	1,1

δ : prob. of one more round.

grim-trigger: No deviation: $3 + 3\delta + 3\delta^2 + \dots = \frac{3}{1-\delta}$
$(1-\delta)(3 + 3\delta + \dots) = 3$

deviation: $(1-\delta)5 + 1\delta$

no dev. if. $(1-\delta)5 + \delta \leq 3$

$$(1-\delta)5 + (1-\delta)\delta + (1-\delta)\delta^2 + (1-\delta)^3 = (1-\delta)5 + \delta$$

[in HD: $(3,0,3,0,\ldots) \hat{=} 3(1-\delta) + 3\delta^2(1-\delta) + \ldots = \frac{3(1-\delta)}{1-\delta^2} = \frac{3}{1+\delta}$ ✓]

no dev: 3,3,3,3,3,
dev: 5,1,1,3,3,...

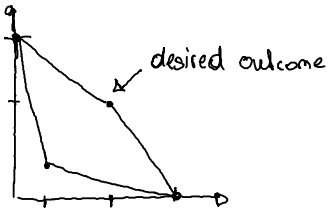
$3 > 7 \Rightarrow \exists\, \delta$ suff. large s.t. no dev.

$3 > (5+\delta+\delta^2)(1-\delta) + 3\delta^3$

⇒ relationships don't require eternal punishment

⇒ game theory gives us great relationship advise

what properties do we need for this strategy?



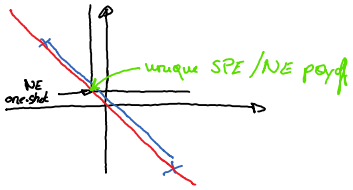
renegotiation proof.

Repetition

Dienstag, 19. April 2016 13:17

∞ - repeated game with discount factor

Zero-sum game



	1/2	1/2
1/2	1, -1	-2, 2
1/2	-2, 2	1, 1

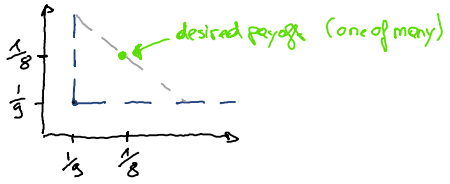
→ $(-\frac{1}{2}, -\frac{1}{2})$

contains feasible points

Cournot competition

$u_i(q) = (1 - q_1 - q_2 - c) q_i$

first: find NE of one-shot game: if $c=0$ $q_1 = q_2 = \frac{1}{3}$. → $u_i = \frac{1}{9}$



$u_1 + u_2 = (1 - q_1 - q_2 - c)(q_1 + q_2)$: if $c=0$ $q_1 = q_2 = \frac{1}{4}$ $u_i = 1/8$

start: $q_1 = \frac{1}{4}$

continue unless $q_1 = \frac{1}{4}$ and q_2 s.t. $u_1(\frac{1}{4}, q_2) < \frac{1}{8}$, then choose $\frac{1}{3}$ forever

q_2, $q_2 = \frac{1}{4}$ s.t. $u_2(q_1, \frac{1}{4}) < \frac{1}{8}$, then choose $\frac{1}{3}$ deviate

Player 2 has the same strategy.

$\frac{1}{8} \geq (1-\delta)\frac{9}{64} + \delta\frac{1}{9}$

$\frac{\partial}{\partial q_1} = (1 - q_1 - q_2) q_1 = 1 - q_2 - 2q_1 \stackrel{!}{=} 0$

$q_1 = \frac{1 - q_2}{2}$

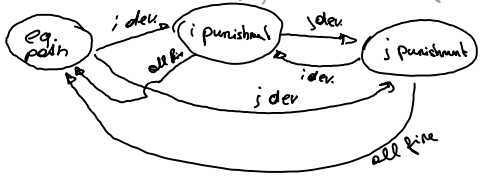
$q_2 = \frac{1}{4}$ → $q_1 = \frac{3}{8}$

$u_1 = (1 - \frac{3}{8} - \frac{1}{4})\frac{3}{8} = \frac{9}{64}$

if $\frac{1}{8} > (1-\delta)\frac{9}{64} + \delta\frac{1}{9}$ then $\exists n$ s.t. punishment for n rounds suffices

∃ harsher punishment: $q_i = 1$ → $u_2 \leq 0$ (0 is minmax payoff : $\max\{1 - q_1 - q_2, 0\}$ as price can't be negative)

write down what both players would do if deviation from punishment



Back at Prisoners Dilemma:

C D

2 possible games each $\frac{1}{2}$

- ends after 10 rounds for sure → the moment we get the news always defect

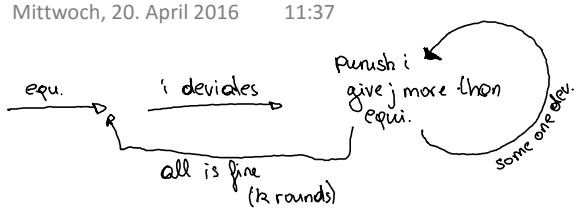
Back at Prisoners Dilemma:

	C	D
C	3,3	0,5
D	5,0	1,1

2 possible games each $\frac{1}{2}$
- ends after 10 rounds for sure → the moment we get the news always defect.
- prob. of δ for every round

Question -hour

Mittwoch, 20. April 2016 11:37



Cournot: deviator $q_i = 0$ $u_i = 0$
punisher $q_j = \frac{1}{2}$ $u_j = \frac{1}{4}$

in eq:
$q_i = q_j = \frac{1}{4}$
$u_i = u_j = \frac{1}{8}$

$$\frac{1}{8} > (1-\delta)\frac{9}{64} + \frac{1}{8}\delta^{k+1}$$

Payoff stream: $\underbrace{\frac{9}{64}, 0, \dots, 0}_{k \text{ times}}, \frac{1}{8}, \dots \quad \frac{1}{8}, \dots$

$$(1-\delta)\frac{9}{64} + (1-\delta)\frac{1}{8}\delta^{k+1} + (1-\delta)\frac{1}{8}\delta^{k+2} + \dots$$

The punisher doesn't want to deviate, because he gets monopoly payoff - best for him.

If the deviator deviates again?

$BR_i\left(\frac{1}{4}\right) = \frac{1}{4}$

$u_i = \left(1 - \frac{1}{2} - \frac{1}{4}\right)\cdot\frac{1}{4} = \frac{1}{16} > 0$

$$\frac{1}{8}\delta^k > (1-\delta)\frac{1}{16} + \frac{1}{8}\delta^{k+1}$$

In all games an equilibrium exists minmax and maxmin are the same.

→ change the game by ignoring the other persons payoff:

$\frac{1}{3}$	2,2	0,0
$\frac{2}{3}$	0,0	1,1

→

2,-2	0,0
0,0	1,-1

so we don't care anymore what the other one does on max or min. In pure strat minmax ≥ maxmin

In the exam: state the concept and solve according to that.

Adding mistakes: SPE already accounts for that by playing the best of every point

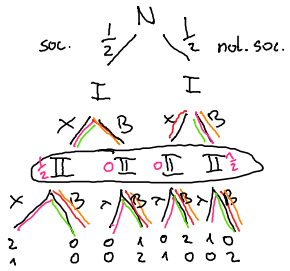
Battle of the sexes when we are not sure of others incentives:

	X	B
X	2,1	0,0
B	0,0	1,2

nature $\frac{1}{2}$ P1 wants to socialize

	X	B
X	0,1	2,0
B	1,0	0,2

$\frac{1}{2}$ P1 hate socialize



$((X,X),X)$ soc n.s. not NE !

$((X,B),X)$ not NE!

$((X,B),B)$ not NE:

$((B,B),B)$ not NE

$((B,X),B)$ only NE!

Auctions: First price auctions.

symmetric distribution, F_i indep. of i
v_i is drawn from F independently

$b_i : R \to R$ my bid/my strategy is a mapping.

$F = \frac{1}{2}[0] + \frac{1}{2}[1]$ $b_i(0) = 0$

~~$b_i(1) = 1$~~ ~~$b_i(1) = 0$~~ → no pure NE!!

$\beta_i : R \to R$ my bid/my strategy is a mapping.

$F = \frac{1}{2}[0] + \frac{1}{2}[1]$ $\beta_i(0) = 0$

~~$\beta_i(1) = 1$~~ ~~$\beta_i(x) = 0$~~ → no pure NE!!

$G_2(x)$ c.d.f.

$\beta_i = b$ (a number)

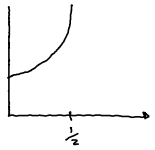
$(1-b)\frac{1}{2}$ + $\frac{1}{2} G_2(b)(1-b)$ = const. = $\frac{1}{2}$ $(b=0)$

what I win: prob. that other has v.0 · prob. that other has v.1 · I win · what I win

$\Rightarrow G_2 = \frac{1}{1-b} - 1$ for $b \leq \frac{1}{2}$

distribution

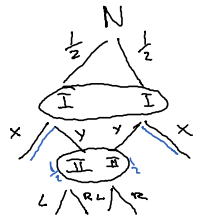
density: $g_2 = \left(\frac{1}{1-b}\right)^2$



Nash equ w/ uncertainty

in the square brackets are lotteries, to show that we cannot multiply

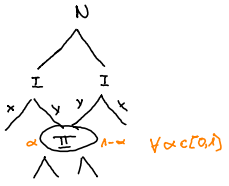
$F = \frac{1}{2}[0] + \frac{1}{2}[1]$ → $x \sim t$



nobody knows the state of the world

in the upper game beliefs have to be $\frac{1}{2}, \frac{1}{2}$

in the lower P1 knows the state, this might change P2 beliefs

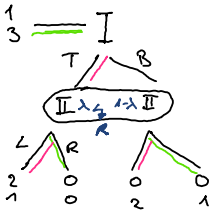


PBE not enough here

(sequentially equilibria ←)

Dienstag, 19. April 2016 14:30

Preperation for tomorrow:



1 subgame

PBE → NE NE ¬ PBE

work down NF game

	L	R
O	1,3	1,3
T	2,1	0,0
B	0,2	0,1

Game Theory with Baysian equilibrium.

Dienstag, 3. Mai 2016 13:17

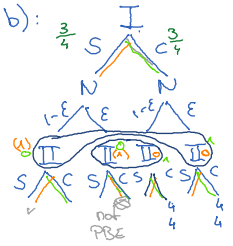
	S	C
S	5,2	3,1
C	6,3	4,4

(S,S) Stackelberg outcome (perfect info)
(C,C) Cournot outcome (simultaneous move)

model a) ε prob. that II does not observe $\Rightarrow$ robust ($\overset{(S,S)}{\text{PBE}}$ outcome) if ε is suff small

b) ε prob that II observes the opposite

game b):



$\varepsilon=0$ → Stackelberg no PBE

→ Cournot PBE ≠ SPE if $\varepsilon=0$

but ∃ PBE s.t. outcome → (5,2) as $\varepsilon \to 0$

$$\frac{\frac{3}{4}(1-\varepsilon)}{\frac{3}{4}\cdot(1-\varepsilon)+\frac{1}{4}\varepsilon}$$

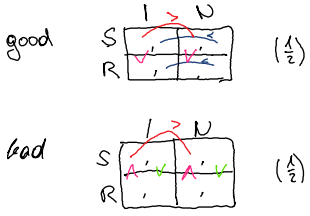
robustness means that it is still an eq. if we change things only slightly

beliefs are influenced by strategies

	S,S	C,C	C,S
S	$(1-\varepsilon)5+\varepsilon\cdot 3$, $(1-\varepsilon)2+\varepsilon\cdot 1$		
C			

normal form

world bank example



if stay the investor needs to better off with intervention.

all equal if good then not intervene

better for the wb if investor stays

(Mi, 4.5. nicht da)

pooling vs. seperating equil

pooling: no information revealed (not,not) (int,int) $(\lambda[int]+(1-\lambda)[not], \lambda[int]+(1-\lambda)[not])$

separating: all information revealed (not,int) (int,not) $[not, \frac{1}{2}[int]+\frac{1}{2}[not])$

partial separating (semi-) rest (?)

"not" good news as $\frac{2}{3} > \frac{1}{2}$

strategies: WB $(\,)_{good,bad} \in \{int,not\}^2$

pooling: (not,not)

strategy: $(\overset{(stay)}{\underset{not}{stay}}\ \underset{int}{run})$ belive that state is bad (good) if intervene.

stay was not BR in HW payoffs.

Signalling and Labeling / Messages

Dienstag, 10. Mai 2016 13:32

Ice cream chain "very good": Ice cream is very bad, because if it was good many shops label themself "very good"
creates a focal point - because it's english
you always have to think about labels without the words

Sender reciever game from homework:

	L	M	R
$\frac{1}{2}\alpha$	a, x	b, y	c, z
$\frac{1}{2}\beta$	e, v	f, u	g, w

$x \geq \max\{y,z\}$
$u \geq \max\{v,w\}$
$a \geq b$
$f \geq e$

Strategies:

I: $m = (\{\alpha\}, \{\beta\})$ (message; state α, β)

II: $a = (L, M)$ ($\{\alpha\}$, $\{\beta\}$)

$\mu_2(\alpha|\{\alpha\}) = 1$, $\mu_2(\beta|\{\beta\}) = 1$ } follows from strategy profile

↑ seperating Equilibrium

now $M = \{\{\alpha\}, \{\beta\}, \{\alpha\beta\}\}$

either α or β, like $\emptyset$ in HW
in that case there is no lying.

$I \equiv I$

II: (L, M, L) ($\{\alpha\}$, $\{\beta\}$, $\{\alpha\beta\}$)

$\mu_2(\alpha|\{\alpha,\beta\}) = 1$ eq. beliefs can be however we want.
just need to be consistent.

II: $(L, M, M) \Leftrightarrow \mu_2(\alpha|\{\alpha,\beta\}) = 0$

Job market signalling

pooling: $(\underset{L}{nd}, \underset{H}{nd})$ $(\underset{nd}{\frac{1}{2}y_L + \frac{1}{2}y_H}, \underset{ed.}{y_L})$ $\mu_2(H|edu) = 0$

back to the messages

	L	M	R
α	a, 3	b, 0	0, 2
β	c, 0	d, 3	0, 2

I $(\{\alpha\beta\}, \{\alpha,\beta\})$ (α, β)

II (R, R, R) $\mu_2(\alpha|\{\alpha,\beta\}) = \frac{1}{2}$ (That follows from equ.)
$\mu_2(\alpha|\{\alpha\}) = \frac{1}{2}$
$\mu_2(\alpha|\{\beta\}) = \frac{1}{2}$ (just don't believe the message)

		a_1	a_2^{*}	a_3^{*}	a_4^{*}	a_5^{*}	
$\frac{1}{4}$	H	1, 0	0, 1*	-1, 0	-2, -1	-3, -2	message {H,h}
$\frac{1}{4}$	h	0, -1	1, 0	0, 1*	-1, 0	-2, -1	
$\frac{1}{4}$	l	-1, -2	0, -1	1, 0	0, 1*	-1, 0	message {L,l}
$\frac{1}{4}$	L	-2, -3	-1, -2	0, -1	1, 0	0, 1*	

II (a_2, a_4) ($\{h,h\}$, $\{l,l\}$)

no seperating equil., in pooling do: a_3 or a_4 or mix

11.3.2016

rules are: i) nature informs P1 privately about state
ii) P1 sends message to P2 message $m \subseteq \{H,h,l,L\}$
iii) P2 makes choice of action M is a subset w/o $\emptyset$ of $\{H,h,l,L\}$

Strategy I: $\{H,h,l,L\} \to M$
Strategy I: $M \to A = \{a_1, a_2, a_3, a_4, a_5\}$

$\nexists$ seperating equilibrium (w/o loss of generality only consider truthtelling)

I σ_1: $\{H,h,l,L\} \to M$ s.t. $\sigma_1(s) = \{s\}$ $\forall s \in \{H,h,l,L\}$
$(\underset{H}{\{H\}}, \underset{h}{\{h\}}, \underset{l}{\{l\}}, \underset{L}{\{L\}})$

II $(\underset{\{H\}}{a_2}, \underset{\{h\}}{a_3}, \underset{\{l\}}{a_4}, \underset{\{L\}}{a_5}, \underset{o/w}{2})$ II is chosing BR, I not $\to$ not an equilibrium

semi-seperating equilibrium

I: $(\underset{H}{\{H,h\}}, \underset{h}{\{H,h\}}, \underset{l}{\{l,L\}}, \underset{L}{\{l,L\}})$

II $(\underset{\{h,H\}}{a_2}, \underset{\{l,L\}}{a_4}, \underset{o/w}{a_2}) = \mu_2(\cdot|m) = \frac{1}{2}[H] + \frac{1}{2}[h]$ s.t. $m \notin \{\{h,H\}, \{l,L\}\}$ or write $\mu_2(H|m) = \frac{1}{2} = \mu_2(h|m)$ $\forall m \notin \{\{H,h\}, \{l,L\}\}$

$I: S \to \{H, h, l, L\}$

$II: M \to a^*$ where a^* BR as if no info, $\mu_2(\cdot|m)$ = signal prior

PBE ∀ payoffs

First price auction bid 1.000€ and win. What do you learn.

ex post I know I bid too much. But we want to look at it ex-ante.

I thought it looked good, but as it is the same price for all, and all the others bid less, the price is apparently wrong. wrong signals.

two quality levels: H, L equally likely (used cars)

buyer sets price (take it or leave it)

$s_H > s_L$ ← value of seller

$b_H > b_L$ ← value of buyer

i) p cannot be set s.t. if buy then quality = H

ii) for mix $p \leq \frac{1}{2} b_H + \frac{1}{2} b_L$

both sell if $p > s_H$, can if $p \geq s_H$

necessary and sufficient condition that $\exists p$ s.t. both can be bought: $s_H \leq \frac{1}{2} b_H + \frac{1}{2} b_L$

iii) can we rule out in which H are never sold? [i.e. there exists a market where H is always sold. → what?]

$s_H < b_L$ $p = s_H$ this price is here because of beliefs

$s_H > b_L$ $\mu_b(L) = 1$ (beliefs only L cars) $p \leq b_L$ as $s_H > b_L$ high-type won't sell → market failure even if $b_H > s_H$ → PBE

$V_S \in U[0,1]$

$V_b = \frac{3}{2} V_s$

$p = ?$

bid p: and buy ~ $p \geq V_S$ so $E(V_S | \text{buy at } p) = \frac{p}{2}$

$E(V_b | \text{buy at } p) = \frac{3}{2} \frac{p}{2} = \frac{3}{4} p$

$p, \frac{3}{4}p, 0$ → don't participate in market because good cars never be good

	buy	not
$\frac{1}{3}$ H	3, 1	0, 0
$\frac{2}{3}$ L	1, -1	0, 0

burning money

	buy	not
H	$3-x_1, 1$	$-x_1, 0$
L	$1-x_2, -1$	$-x_2, 0$

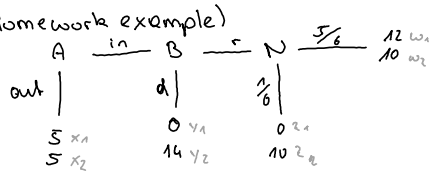
never burn more than 1 in both types

market failure, solve by advertising. → HW

Signalling (national anthem)

Sonntag, 29. Mai 2016 09:51
(Dienstag, 24. Mai 2016

(Homework example)



2 players: A, B $a_A \in \{in, out\}$ publicly

then B $a_B \in \{roll, d\}$

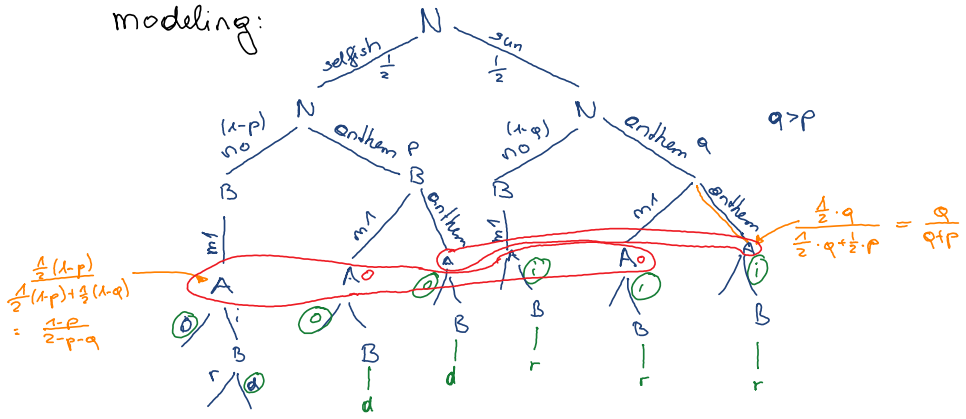
outcomes: 4

utilities: x, y, w, z

Should B talk? How? If everyone cares about their own money unique outcome is (5,5)

Singing the national anthem could signal being a nice person

modeling:



B has no nodes!
A has 2

coordination game.

	L	R
T	1,1	0,0
B	0,0	2,2

without communication we can predict: (T, L) (B, R) $\left(\frac{2}{3}[T] + \frac{1}{3}[B], \frac{2}{3}[L] + \frac{1}{3}[R]\right)$

communication. I sends a message about her action. $\{T\}, \{B\}, \{T, B\}$

II listens

play

ex I: ($\{T\}$, B, T, mixed) m: $\{B\}$ $\{T\}$ $\{T, B\}$ ex II: ($\{B\}$, $\{T\}$, $\{T, B\}$)

$((\{B\}, B, T, B), (R, L, R))$ SPE, PBE we love that

eg. that gets (1,1):

$((\{T\}, T, T, T), (L, L, L))$ SPE, but I lies (we don't like lying)

truthful

$((\{B\}, B, T, \cdot), (R, L, \cdot))$ if I is believable → (2,2) ; we cannot be believable AND get (1,1)

	X	B
X	3,1	0,0
B	0,0	1,3

allowing I to call II always ends in (x,x)

	L	M	R
$(\frac{1}{2})\ \alpha$	3,3	0,0	2,2
$(\frac{1}{2})\ \beta$	0,0	3,3	2,2

predicted outcomes are (L_α, M_β) $(R_{\alpha\beta})$

2 entire strategies:

$((\{\alpha\}, \{\beta\}), (L, M, \quad))$
$\quad\ \alpha \quad \beta \quad\quad \{\alpha\}\ \{\beta\}\ \{\alpha,\beta\}$

$((\{\alpha,\beta\}, \{\alpha\beta\}), (L, M, R))$ I don't want to get (2,2) I will always say sth.

change the game to:

	L	M	R
α	1,3	0,0	2,2
β	0,0	1,3	2,2

$((\{\alpha,\beta\}, \{\alpha\beta\}), (L, M, R))$ → get (2,2) can't get (1,3)

	L	M	R
α	-100,3	0,0	2,2
β	0,0	3,3	2,2

$((\ ?\ ,\ ), (L, M, R))$ don't find a good solution

Reputation

Mittwoch, 25. Mai 2016 11:46

	not enter	enter acco	enter fight
(1-b) weak	2, 0	0, $\frac{1}{2}$	-1, $-\frac{1}{2}$
b strong	2, 0	$-\frac{1}{2}, \frac{1}{2}$	0, $-\frac{1}{2}$

I pure strategies: $M_1 \times A_1$ "red"

$M_1 \times A_1^M$

$A_1^M \Leftrightarrow M \to A_1$ $\quad (m, a_{\{T\}}, a_{\{B\}}, a_{\{T,B\}})$

a strategy is truthtelling if $a_m \in m$ — can be only for sent messages / or for all messages

m is a message if $m \subseteq A_1$ (action played) or $m \subseteq S$ (states) s.t. $m \neq \emptyset$

m is precise if $m = \{a_1\}$ for some a_1

m is rogue if m is not precise

truthtelling with states: if $s \in m_s \ \forall s \in S$

I pure strategies: M^S

(m_α, m_β) $\quad (\{\alpha\}, \{\alpha\})$ sometimes lying, $(\{\beta\}, \{\alpha\})$ always lying.

belief $\mu: M \to \Delta S$

$\mu(m)$ belief if hear $\{\alpha\}$, distribution

listener believing if $\mu_{\{\alpha\}} = [\alpha]$, $P(s=\alpha \mid \{\alpha\}) = 1$ or $\mu_{\{\beta\}}(\alpha) = 1$

$\mu_{\{\beta\}} = [\beta]$, $P(s=\beta \mid \{\beta\}) = 1$ $\quad \mu_{\{\beta\}}(\beta) = 1$

$\mu_{\{\alpha,\beta\}}$ no restrictions

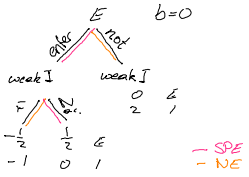
listener believing for 2x2 game if $a_{\{T\}} = T$, $a_{\{B\}} = B$, $a_{\{T,B\}}$ = no restrictions

Reputation

chain store < fight / accommodate

n cities

1 entrant per city < enter / not



Single city n=1

not enter: 0

enter: $(1-b)\frac{1}{2} + b(-\frac{1}{2}) = \frac{1}{2} - b \quad b < \frac{1}{2} \Rightarrow$ enter

Assume $b < \frac{1}{2}$

two cities; n=2

E_1: {not, enter}

E_2: {E_1 not, E_1 enter F, E_1 enter A} → {not, enter}

I weak: E_1 enter → {F, A}

E_1 not, E_2 enter → {F, A}

E_1 enter F, E_2 enter → {F, A}

E_1 enter A, E_2 enter → {F, A}

E_1 not enter: enter, I weak A

E_2 enters, always A_1: enter, I weak A

E_1 enters, always F_1: enter, I weak A

E_1 enters A_1 w/prob $1-\alpha$:

E_2 enters w/prob λ after E_1 enters F_1

E_1 enters: I weak if $A_1 = 0 + 0$

$F_1 = -1 + (1-\lambda)2 + \lambda \cdot 0 = 1 - 2\lambda \Rightarrow \lambda = \frac{1}{2}$ to make indiff

E_2 after F_1: if not 0

if enter weak w/prob. $\frac{\alpha(1-b)}{\alpha(1-b)+b} = \frac{1}{2}$

Reputation/chain store/

Dienstag, 31. Mai 2016 13:19

$V_S \sim G$

$V_B = f(V_S)$ f strictly increasing, $f(v_S) > v_S$

p?

Q: under what conditions is $p=0$ the only optimal price? ($\rightarrow$ so optimal payoff/$u = 0$)

start with $p > 0$

$u(p) = -p + E(f(V_S) \mid V_S \le p) < 0$

$p > \int_0^p f(v_S) \frac{g(v_S)}{G(p)} dv_S$ $\forall p > 0$

$E(h(X) \mid X \le y) = \int_0^y h(x) \cdot \frac{g(x)}{G(x)} dx$ $X \sim G$ (resires w/ sums)

$E(h(X) \mid X \le y) = \sum_{i : v_S(i) \le y} f(v_S(i)) P(V_S = v_S(i) \mid v_S(i) \le y)$

$P(V_S = v_S(i)) / P(v_S(j) \le y)$

Example $G = [0,1]$ $f(x) = \frac{3}{2}x$

Chain store paradox, revisited

1 city: enter if $b < \frac{1}{4}$

2 cities: E_1 enter if $b < \frac{1}{4}$ $\rightarrow$ E_2 enter if E_1 not, $P(E_2 \text{ enter}) = \frac{1}{2}$ if E_1 enter + E_1, E_2 enter if E_1 enter + A_1

E_1 not: $\frac{1}{4} < b < \frac{1}{2}$ $\rightarrow$ E_2 enter, E_2 enter

('left eq. could work down now; need beliefs')

n cities: given $b > 0$ F_k for k not too large

E_k no entry for k not too large

Herding

2 restaurants A+B $P(A>B) = 0.51$

n people, all the same taste

sequentially visit rest + observe visit choice of previous

before visit each gets signal about best restaurant; true w/ prob. $b > \frac{1}{2}$

1st person: signal $S_1 = B$: $P(A>B \mid S_1 = B) = \frac{P(S_1 = B \mid A>B) \cdot P(A>B)}{\text{---''---} \; P(S_1 = B \mid B>A) \cdot P(B>A)} = \frac{(1-b) \cdot 0.51}{(1-b) 0.51 + b \cdot 0.49} < \frac{1}{2}$ if b is suff large. (Little more than $\frac{1}{2}$)

$\rightarrow$ follows if B suff. large

if 2nd person get A when I want to B $\rightarrow$ go to A because prob

2nd: B_1

Buyer seller - thing

Mittwoch, 1. Juni 2016 11:33

Buyer - seller

Buyer: $p \in \mathbb{R}_+$

seller: $\mathbb{R}_+ \times \mathbb{R}_+ \to \{yes, no\}$ yes if $p > v_s$

	v_1	v_2	v_3
p_1	x_{11}	x_{12}	x_{13}
p_2			

x_{11}: if $v_1 \leq p$: $\underbrace{f(v_1)}_{v_b} - p = u_B(p, v_1)$

$v_1 > p : 0 = u_b$ (not bought)

$$u(p_1, .) = \sum x_{1i} Pr(v = v_i) = \sum_{i: p_1 \geq v_i} x_{1i} Pr(v = v_i) = \left[\sum_{i: p_1 \geq v_i} P(V = v_i) f(v_i)\right] - p_1 \cdot P(p_1 > V)$$

V is random variable that determines v_i

Restaurant example again.

b: P(correct signal) $b > \frac{1}{2}$ suff. large

observe $A_1, b_2 \to A_2$ if $P(A > B \mid A_1, b_2) > \frac{1}{2}$

$u_2(A_2 \mid A_1, b_2) = 1 \cdot P(A > B \mid A_1, b_2) + 0 \cdot P(B > A \mid A_1, b_2)$

$u_2(B_2 \mid A_1, b_2) = 1 \cdot P(B > A \mid A_1, b_2) + 0 \, P(A > B \mid A_1, b_2)$

A_2 if $u_2(A_2 \mid \ ) > u_2(B_2 \mid \ ) \Longleftrightarrow$

$$(R, \overbrace{s_1 \ldots s_n}^{n+1 \text{ stages}})$$

$R \in \{A, B\}$ $s_1 \in \{A, B\}$

1: $a_1 \to A_1, b_1 \to B_1$ (b suff large)

2: $(A_1, a_2) \to A_2$
$(A_1, b_2) \to A_2$ as if no info
$(B_1, b_2) \to B_2$
$(B_1, a_2) \to A_2$ as if no info.

$$P(B > A \mid b_1) = \frac{b\, 0.49}{b\, 0.49 + (1-b)\, 0.51} > \frac{1}{2} \text{ if } b \text{ is suff large}$$

$b > 0.51$

$$P(A > B \mid B_1 a_2) = \frac{0.51 \cdot (1-b) b}{0.51 (1-b) b + 0.49 \cdot b (1-b)} = 0.51$$

3: $A_1 A_2 * \to A_3$

$B_1 A_2 a_3 \to A_3$
$B_1 A_2 b_3 \to B_3$

$B_1 B_2 * \ldots\ldots * \to B_k$ $\quad P(B_1 = \ldots = B_k \mid A > B) = (1-b)^2 \lessapprox \frac{1}{4}$

$A_1 x \ldots\ldots * \to A_k$ $\quad P(A_1 = \ldots A_k \mid B < A) = 1 - b \leq \frac{1}{2}$

P(ineff in long run) $\gtrsim \frac{1}{4} \cdot 0.51 + \frac{1}{2} \cdot 0.49$ if $b \sim 0.51$

Repetition - Bargaining

Dienstag, 21. Juni 2016 13:37

a) take it or leave it - one round

b) alternating offers

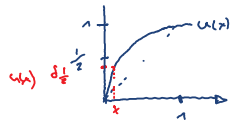
(Stift leer, continued on Wed June 22

Random proposer

2 rounds, divide dollar, risk neutral

	1 prop	exp	
2nd round	$(1,0)$	$(\frac{1}{2},\frac{1}{2})$	$1-\delta+\delta^2-\delta^3+\dots+(-1)^k\delta^k$
1st round	$(1-\frac{\delta}{2},\frac{\delta}{2})$	$(\frac{1}{2},\frac{1}{2})$	

"introduce" utility: $u(0)=0$ $u(1)=1$ $u(x)=\sqrt{x}$ (exp. utility)

2nd round: $(1,0)$ $(\frac{1}{2},\frac{1}{2})$

1st round: $(1-x,x)$ s.t. $u(x)=\frac{\delta}{2} \Rightarrow x=u^{-1}(\frac{\delta}{2})$